C O M M O N W E A L T H O F A U S T R A L I A

DEPARTMENT OF NATIONAL DEVELOPMENT

DIVISION OF NATIONAL MAPPING

TECHNICAL REPORT No. 2

Report on the Khancoban azimuth test of the accuracy obtainable with Wild T4 and T3 theodolites

by

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Canberra A.C.T., Australia March 1963

REPORT ON THE KHANCOBAN AZIMUTH TEST OF THE ACCURACY OBTAINABLE WITH WILD T4 and T3 THEODOLITES

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Summary: The aim of this report is to derive a value for the absolute error of an astronomical azimuth determined with a Wild T4 Astro-Theodolite or a Wild T3 Theodolite by method of reading the angle between Sigma Octantis and a terrestrial reference object on six zeroes.

Introduction: The observations analysed in this paper were made at Khancoban in November 1961 by officers of the Division of National Mapping and the Snowy Mountains Authority. The result of the observations are listed in Appendix A.

The test line near Khancoban, N.S.W., in the south-western part of the Snowy Mountains Area is 11 miles long, running well clear off the ground between trig stations Cochrane Gap and Scammels Lookout, both of which are situated on high hills. The observations were carried out in excellent visibility at mild temperatures generally with a slight breeze.

Nine observers with two Wild T4 Theodolites and four Wild T3 Theodolites participated in the test. Observations were made over a period of five consecutive nights. On the first three nights, simultaneous observations were taken from several eccentric positions at Cochrane Gap involving Two T4's and four T3's. On the last two nights, simultaneous reciprocals were taken with one T4 and one or two T3's at either end of the line. On all occasions every precaution was taken to reduce controllable errors to a minimum.

In February 1962 the azimuth of the line was re-observed from either end with a Wild T4 Astro-Theodolite.

All observations have been corrected for the following -

1. Stride level (T4) or plate bubble (T3) correction.

2. Curvature correction for the time interval between FL and FR pointings to Sigma Octantis.

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3. Reduction to centre.

4. Convergence of meridian between ecce and centre.

5. Reduction to the mean pole.

6. Diurnal aberration.

7. Skew normals.

8. Deviation of the vertical.

9. Laplace correction (A - G) Sin ϕ

In addition to the above all observations taken from Scammels Lookout have been reduced to read as from Cochrane Gap to Scammels Lookout by applying the meridian convergence which has been computed from geodetic longitudes and is regarded to be correct to 0".01.

Due to the standard errors in the T4 observed astro longitudes of $\pm$ 0".24 at Cochrane Gap and $\pm$ 0".13 at Scammels Lookout, the Laplace corrections at these stations have standard errors as follows -

Cochrane Gap : $\pm$ 0".24 Sin ϕ = $\pm$ 0".14

Scammels Lookout : $\pm$ 0".13 Sin ϕ = $\pm$ 0".08

The common corrections to the observations listed separately for the two ends of the line are -

	Cochrane	Scammels
Skew normals	- ".08	- ".04
Deviations of the vertical	+ .37	+ .56
Diurnal aberration	- .32	- .32
Laplace correction	- 12.11	- 16.92
Geodetic convergence		+ 235.90

Station Data:

Cochrane Gap Elevation 1814 ft

	ϕ	λ
Astro	- 36° 10'31".08	- 148° 04' 04"51 $\pm$ 0"24
Geodetic	34".67	25".02

Scammels Lookout Elevation 3724 ft

Astro	- 36° 18'19".83	- 148° 10'35".46 $\pm$ 0".13
Geodetic	24".72	64".04

Analysis of the Observations:

All observations as shown in Appendix A have been included in the computation of a weighted mean azimuth which can be assumed to be very close to the true azimuth of the line. Weights have been allocated according to the number of zeroes taken, the unit weight being equivalent to six zeroes (One zero = one F.L. and one FR pointing). Table 1 in the Appendix shows the computations leading to the mean of all observations. Table 2 lists all T4 observations only whereas Table 3 deals with all T3 observations. The condensed results of Tables 1 to 3 are as follows -

	Mean Azimuth $145^{\circ}30'$ +	Weight of mean	s.e. of mean	s.e. of unit weight = 6 zeroes	Range of observations
All observations	57".72	57	± 0".15	± 1.12	4".38
T4 only	57".82	30	± 0".19	± 1".05	2".29
T3 only	57".62	27	± 0".23	± 1".18	4".38

The mean azimuth of all T3 sets is only 0".20 different from the mean of all T4 sets. The standard error of an observation of unit weight taken with a T3 is ± 1".18 only ± 0.13 more than the corresponding T4 value.

The observation series in Tables 2 and 3 have been observed as follows -

Table 2 : By four observers with two T4 instruments on nine different nights from both ends of the line resulting in reciprocal and simultaneous reciprocal observations. Total number of zeroes = 180.

Table 3 : Five observers on five different nights with four T3 instruments from both ends of the line, resulting in reciprocal and simultaneous reciprocal observations. Total number of zeroes = 162.

The standard errors of the two series referring to a set of six zeroes are regarded as the external errors of such sets.

The corresponding internal error of a set of six zeroes is computed from the individual six observations. In our case we will derive this error from the range (spread) of the sets in order to reduce the labour of computing. With the aid of Tippet's table of the mean normal range we can estimate the standard error. Finally, we compute the average of the standard errors taken with the same type of instrument. See Tables 4 and 5.

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In Tables 2 and 4 (T4) as well as in Tables 3 and 5 (T3) we have standard errors for the same quantity although obtained from samples of different size. Both standard errors are in terms of the mean of six zeroes derived, however, in the one case (Tables 4 and 5) from the individual observations which make up the mean resulting in the internal error, and in the other case from all such mean values obtained with the same type of instrument resulting in the external error.

Summary of standard errors

	Internal error	External error		
Type of instrument	Average s.e. of the mean of six zeroes	s.e. of the mean of a set of six zeroes from the mean of all sets	Ratio	External / Internal
Wild T4	± 0".35	± 1".05		3 to 1
Wild T3	± 0".54	± 1".18		2.2 to 1

Again we see that T4 and T3 observations have standard errors of a similar magnitude giving emphasis to a growing suspicion that T3 azimuths are almost equivalent to T4 azimuths provided that the astro longitude is known with sufficient accuracy in the case of T3 observations and that the observed lines are in latitudes similar to Khancoban.

The summary also shows that the external error of a set of six zeroes is between 2 and 3 times larger than the corresponding internal error.

As most of the azimuth observations in Australia are carried out in an average of two nights (T4) and three nights (T3) and by one observer with one instrument from one end of the line the resulting standard errors can be regarded as internal errors. Analogous of the Khancoban results their to be expected external errors are between 2 and 3 times larger than their internal errors.

Analysis of available Laplace Azimuths on the Australian mainland in view of the Khancoban results.

The average internal standard error of the mean of six zeroes of 113 Wild T4 Azimuths as well as of 29 Tavistock and T3 azimuths has been computed from azimuth abstracts of 142 Laplace stations. The results are as follows -

	T4	Tavistock or T3
No. of stations	113	29
No. of zeroes observed	3122	1360
Average number of nights	2	3
Min. s.e. of six zeroes	± 0.10	± 0.52
Max. s.e. of six zeroes	± 1.14	± 0.92
Average s.e. of six zeroes	± 0.56	± 0.68

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Based on the average s.e. of the mean of six zeroes we can expect the following internal and external errors for the mean of 24 zeroes:

Instrument	Internal		External
	s.e. of six z.	s.e. of 24 z.	s.e. of 24 z.
Wild T4	± 0".56	± 0".28	± 0".6 to ± 0".9
Tavistock or Wild T3	± 0".68	± 0".34	± 0".7 to ± 1".0

Conclusion:

The Khancoban test as well as the results of 142 Laplace Azimuths show that Wild T3 or Tavistock azimuths are nearly as accurate as Wild T4 azimuths. It seems justified to incorporate more T3 and Tavistock azimuths in control traverses in conjunction with longitude observations. It can be assumed that the Kern DKM3 will give similar results.

On a normal line similar to the one in Khancoban, it can be expected that the external error (standard deviation from the unknown true azimuth) of the mean of a small number of sets of six zeroes is between two and three times the corresponding internal standard error.

OBSERVATIONS FROM COCHRANE GAP TO SCAMMELLS LOOKOUT : 145° 30' +

Date	9 Nov 61	10 Nov 61	11 Nov 61	12 Nov 61	13 Nov 61
Instrument	T4/56091	T4/56091	T4/56091	T4/56091	T4/56091
Observer	A	A	A	A	A
Time	2030-2330	1920-2125	1926-2126	1920-2135	1927-2122
No. Zeroes & Range	8(2".14)	11(3".77)	12(3".11)	13(1".82)	12(2".63)
Mean	58.84	58.59	58.63	58.86	59.12
Instrument	T4/37448	T4/37448	T4/37448		
Observer	B	B	B		
Time	2000-2300	1920-2100	1930-2115		
No. Zeroes & Range	12(2".31)	12(1".42)	12(2".79)		
Mean	57.66	57.75	57.92		
Instrument	T3/26687	T3/18516	T3/18788		T3/29886
Observer	C	C	C		C
Time	2006-2050	1922-2003	2035-2124		2044-2134
No. Zeroes & Range	6(2".35)	6(4".07)	6(3".30)		6(2".84)
Mean	58.79	57.53	59.12		58.95
Instrument	T3/26687	T3/18516	T3/18788		T3/29886
Observer	D	D	D		D
Time	2235-2315	2025-2100	1930-2013		1940-2022
No. Zeroes & Range	6(2".41)	6(2".93)	6(5".89)		6(2".94)
Mean	57.21	56.62	56.54		55.25
Instrument	T3/29886	T3/29886	T3/18516	T3/26687	
Observer	E	E	E	E	
Time	2004-2048	1920-2003	2035-2124	1923-2012	
No. Zeroes & Range	6(3".23)	6(4".02)	6(2".38)	6(2".84)	
Mean	57.05	57.02	57.31	58.68	
Instrument		T3/29886	T3/18516	T3/26687	
Observer		F	F	F	
Time		2024-2100	1930-2015	2036-2130	
No. Zeroes & Range		6(2".08)	6(3".55)	6(1".92)	
Mean		57.15	56.84	57.24	
Instrument		T3/26687	T3/26687	T3/18788	
Observer		G	G	G	
Time		1920-2100	1930-2123	1920-2126	
No. Zeroes & Range		12(4".73)	12(4".95)	12(6".23)	
Mean		56.81	58.78	59.63	
Instrument		24.2.62 T4/56091	25.2.62 T4/56091		
Observer		K	K		
Time		1905-2110	1905-2010		
No. Zeroes & Range		23(4".21)	13(4".11)		
Mean		57.07	57.74		

OBSERVATIONS FROM SCAMMELLS LOOKOUT TO COCHRANE GAP

Reduced to read as from Cochrane Gap to Scammells Lookout

	12.11.61	13.11.61	22.2.62	23.2.62
Instrument	T4/37448	T4/37448		
Observer	B	B		
Time	2035-2130	1940-2130		
No. Zeroes & Range	6(1"35)	12(1"83)		
Mean	57.34	57.44		
Instrument	T3/18516			
Observer	C			
Time	1927-2014			
No. Zeroes & Range	6(4"06)			
Mean	57.51			
Instrument	T3/18516			
Observer	D			
Time	2034-2126			
No. Zeroes & Range	6(2"93)			
Mean	58.23			
Instrument		T3/18516		
Observer		E		
Time		2045-2132		
No. Zeroes & Range		6(3"20)		
Mean		57.11		
Instrument		T3/18516		
Observer		F		
Time		1938-2025		
No. Zeroes & Range		6(1"92)		
Mean		56.31		
Instrument		T3/18788		
Observer		G		
Time		1938-2130		
No. Zeroes & Range		12(4"78)		
Mean		57.41		
Instrument	T4/37448			
Observer	H			
Time	1926-2030			
No. Zeroes & Range	6(1"32)			
Mean	57.47			
Instrument			T4/56091	T4/56091
Observer			K	K
Time			1900-2100	1900-1955
No. Zeroes & Range			20(4"52)	12(1"51)
Mean			56.83	57.34

SUMMARY OF WEATHER CONDITIONS

9th Nov 61 Cool, clear, light Westerly wind
10th Nov 61 Cool, some cloud, Westerly breeze
11th Nov 61 Cool, clear, calm first half of obs. then high S.E. breeze.
12th Nov 61 Mild, calm, clear
13th Nov 61 Mild, some cloud, S.E. breeze.

The following is a Summary by Instruments of the Wild T3 results at Cochrane Gap

T3 Inst. No.	26687	18516	18788	29886
9th Nov 61	58.79 : 57.21			57.05
10th Nov 61	56.81	57.53 : 56.62		57.02 : 57.15
11th Nov 61	58.78	57.31 : 56.84	59.12 : 56.54	
12th Nov 61	58.68 : 57.24		59.63 :	
13th Nov.61				58.95 : 55.25
Overall Means	57.89	57.08	58.73	57.08

Wild T4 results at Cochrane Gap

T4 Inst. No.	37448	56091
9.11.61	57.66	58.84
10.11.61	57.75	58.59
11.11.61	57.92	58.63
12.11.61		58.86
13.11.61		59.12
24. 2.62		57.07
25. 2.62		57.74
Mean	57.78	58.41

Summary by Instruments at Scammells Lookout

Wild T3

T3 Inst.No	18516	18788
12.11.61	57.51:58.23	
13.11.61	57.11:56.31	57.41
Mean	57.29	57.41

Wild T4

T4 Inst. No.	37448	56091
12.11.61	57.34:57.47	
13.11.61	57.44	
22. 2.62		56.83
23. 2.62		57.34
Mean	57.41	57.02

TABLE 1

List of all Observations T4 and T3

Observer	Weight P	Observed Mean 145° 30' +	Residual V	pv
A T4	1	58.84	- 1.12	- 1.12
	2	8.59	- .87	- 1.64
	2	8.63	- .91	- 1.82
	2	8.86	- 1.14	- 2.28
	2	9.12	- 1.40	- 2.80
B T4	2	57.66	+ .06	+ .12
	2	7.75	- .03	- .06
	2	7.92	- .20	- .40
	1	7.34	+ .38	+ .38
	2	7.44	+ .28	+ .56
C T3	1	58.79	- 1.07	- 1.07
	1	7.53	+ .19	+ .19
	1	9.12	- 1.40	- 1.40
	1	8.95	- 1.23	- 1.23
	1	7.51	+ .21	+ .21
D T3	1	57.21	+ .51	+ .51
	1	6.62	+ 1.10	+ 1.10
	1	6.54	+ 1.18	+ 1.18
	1	5.25	+ 2.47	+ 2.47
	1	8.23	- .51	- .51
E T3	1	57.05	+ .67	+ .67
	1	7.02	+ .70	+ .70
	1	7.31	+ .41	+ .41
	1	8.68	- .96	- .96
	1	7.11	+ .61	+ .61
F T3	1	57.15	+ .57	+ .57
	1	6.84	+ .88	+ .88
	1	7.24	+ .48	+ .48
	1	6.31	+ 1.41	+ 1.41
G T3	2	56.81	+ .91	+ 1.82
	2	8.78	- 1.06	- 2.12
	2	9.63	- 1.91	- 3.82
	2	7.41	+ .31	+ .62
H T4	1	57.47	+ .25	+ .25
K T4	3	56.83	+ .89	+ 2.67
	2	57.34	+ .38	+ .76
	4	7.07	+ .65	+ 2.60
	2	7.74	- .02	- .04

$\Sigma p = 57$

$\Sigma +pv = +21.17$
$\Sigma -pv = -21.27$
$\Sigma pvv = 46.784$

Weighted mean $= \frac{440.24}{57} = 7.72$

Standard error of weighted mean $= \pm \sqrt{\frac{46.784}{57 \times 37}} = \pm 0''.15$

Weighted mean : $57''.72 \pm 0''.15$ (s.e.)

Standard error of unit weight $= \pm \sqrt{\frac{46.784}{37}} = \pm 1''.12$

TABLE 2

T4 Observations only

w	Observer	Weight P	Observations o	Residual v	pv
1	A	1	58.84	- 1.02	- 1.02
2		2	8.59	- 0.77	- 1.54
3		2	8.63	- 0.81	- 1.62
4		2	8.86	- 1.04	- 2.08
5		2	9.12	- 1.30	- 2.60
6	B	2	57.66	+ 0.16	+ 0.32
7		2	7.75	+ 0.07	+ 0.14
8		2	7.92	- 0.10	- 0.20
9		1	7.34	+ 0.48	+ 0.48
10		2	7.44	+ 0.38	+ 0.76
11	H	1	57.47	+ 0.35	+ 0.35
12	K	3	56.83	+ 0.99	+ 2.97
13		2	7.34	+ 0.48	+ 0.96
14		4	7.07	+ 0.75	+ 3.00
15		2	7.74	+ 0.08	+ 0.16

$\Sigma p = 30 \quad \Sigma po = 234.52$

$\Sigma + pv = 9.14$
$\Sigma - pv = 9.10$
$\Sigma \quad pv = +0.04$
$\Sigma \quad pvv = 15.4586$

Weighted mean $= \frac{234.52}{30} = 57''.82 \pm 0''.19$

Standard error of unit weight $= \pm\sqrt{\frac{15.4586}{14}} = \pm \ 1''.05$

Standard error of weighted mean $= \pm \frac{1.05}{\sqrt{30}} = \pm \ 0''.19$

TABLE 3

T3 Observations only

	Observer	Weight P	Observations o	Residual v	pv
1	C	1	58.79	- 1.17	
2		1	57.53	+ 0.09	
3		1	59.12	- 1.50	
4		1	58.95	- 1.33	
5		1	57.51	+ 0.11	
6	D	1	57.21	+ 0.41	
7		1	56.62	+ 1.00	
8		1	56.54	+ 1.08	
9		1	55.25	+ 2.37	
10		1	58.23	- 0.61	
11	E	1	57.05	+ 0.57	
12		1	57.02	+ 0.60	
13		1	57.31	+ 0.31	
14		1	58.68	- 1.06	
15		1	57.11	+ 0.51	
16	F	1	57.15	+ 0.47	
17		1	56.84	+ 0.78	
18		1	57.24	+ 0.38	
19		1	56.31	+ 1.31	
20	G	2	56.81	+ 0.81	+ 1.62
21		2	58.78	- 1.16	- 2.62
22		2	59.63	- 2.01	- 4.02
23		2	57.41	+ 0.21	+ 0.42

$\Sigma p = 27 \quad \Sigma po = 205.72$

$\Sigma +$ pv = 12.03
$\Sigma -$ pv = 12.01
Σ pv = +0.02
Σ pvv = 30.7518

$$\text{Mean} = \frac{\Sigma po}{\Sigma p} = 57.62 \pm 0''.23$$

$$\text{s.e. of unit weight} = \sqrt{\frac{\Sigma pvv}{n-1}} = \pm \sqrt{\frac{30.7578}{22}} = \pm 1''.18$$

$$\text{s.e. of mean} = \pm \frac{1''.18}{\sqrt{27}} = \pm 0''.23$$

TABLE 4

Determination of the internal standard error of a set of six zeroes from the range of observations

Wild T4 Instrument

(a) from 10 sets of 12 zeroes

	Range		Range
1	3"77	6	1"42
2	3.11	7	2.79
3	1.82	8	4.11
4	2.63	9	1.83
5	2.31	10	1.51
		Sum	25.30

Standard error of a single observation = $\frac{2.53}{3.258} = \pm$ 0"78

Standard error of the mean of six zeroes $= \pm$ 0"78 $\sqrt{\frac{1}{6}}$ = 0"32 (p = 20)

(b) from 2 sets of six zeroes

	Range
1	1"35
2	1.32
Sum	2.67

Standard error of the mean of six zeroes

$= \pm \frac{1.33}{2.534} \sqrt{\frac{1}{6}} = \pm$ 0"21 (p = 2)

(c) from 1 set of 8 zeroes

s.e. of the mean of six zeroes $= \pm \frac{2.14}{2.847} \sqrt{\frac{1}{6}} = \pm$ 0"31 (p = 1)

(d) from 1 set of 20 zeroes

s.e. of the mean of six zeroes $= \pm \frac{4.52}{3.735} \sqrt{\frac{1}{6}} = \pm$ 0"49 (p = 3)

(e) from one set of 24 zeroes

s.e. of the mean of six zeroes $= \pm \frac{4.21}{3.86} \sqrt{\frac{1}{6}} = \pm$ 0"45 (p = 4)

Average standard error a - e $= \pm$ 0"35

TABLE 5

Determination of the internal standard error of a set of six zeroes from the range of observations

Wild T3 Instrument

(a) From 19 sets of six zeroes

	Range		Range
1.	2".35	11	2".38
2.	4".07	12	2".84
3.	3".30	13	2".08
4.	2".84	14	3".55
5	2".41	15	1".92
6	2".93	16	4".56
7	5".89	17	2".93
8	2".94	18	3".20
9	3".23	19	1".92
10	4".02		
		Sum	59".36

Standard error of a single observation $= \pm \frac{59.36}{19 \times 2.534} = \pm$ 1".23

Standard error of the mean of six zeroes $= \pm$ 1".23 $\sqrt{\frac{1}{6}} = \pm$0"50(p=19)

(b) From 4 sets of 12 zeroes

	Range
1	4".73
2	4".95
3	6".23
4	4".78
Sum	20"69

Standard error of the mean of six zeroes $= \pm \frac{20.69}{4 \times 3.258} \sqrt{\frac{1}{6}} = \pm$ 0"65(p=8)

Average standard error a - b = $\pm$ 0".54